

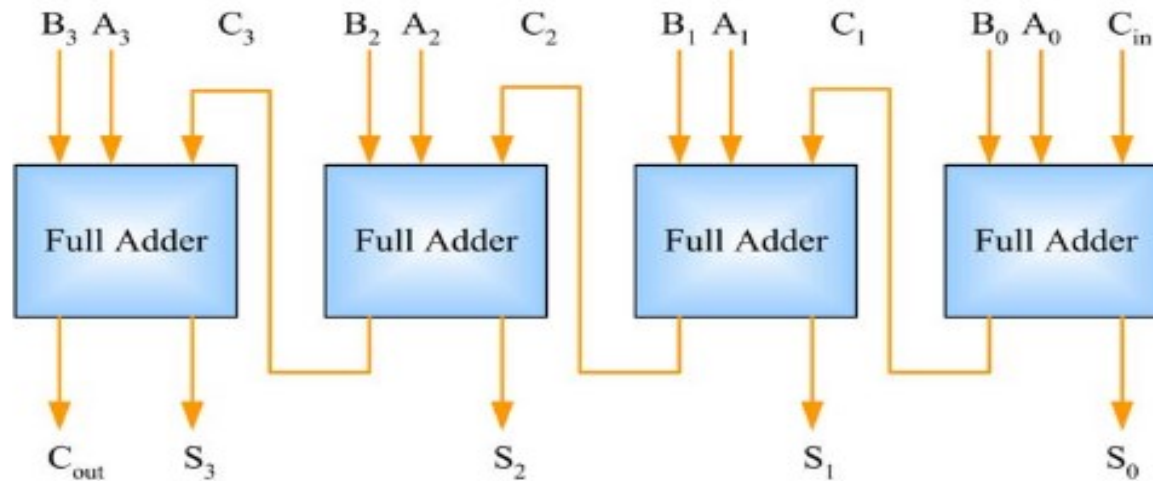
# Combinational vs Sequential

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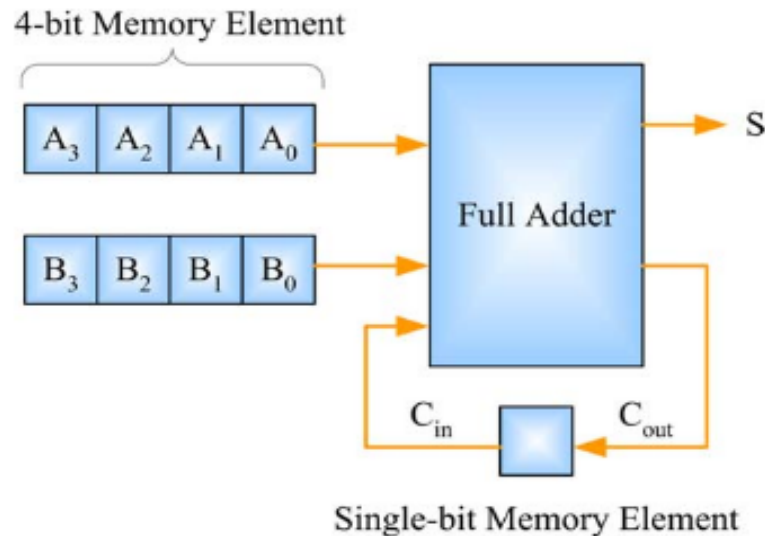
- ✓ A combinational circuit:
  - At any time, **outputs depends only on inputs**
    - Changing inputs changes outputs
  - No regard for previous inputs
    - **NO MEMORY** (history)

# Combinational Adder



- ✓ 4-bit adder (ripple-carry)
  - Notice how carry-out propagates
  - One adder is active at a time

# Sequential Adder

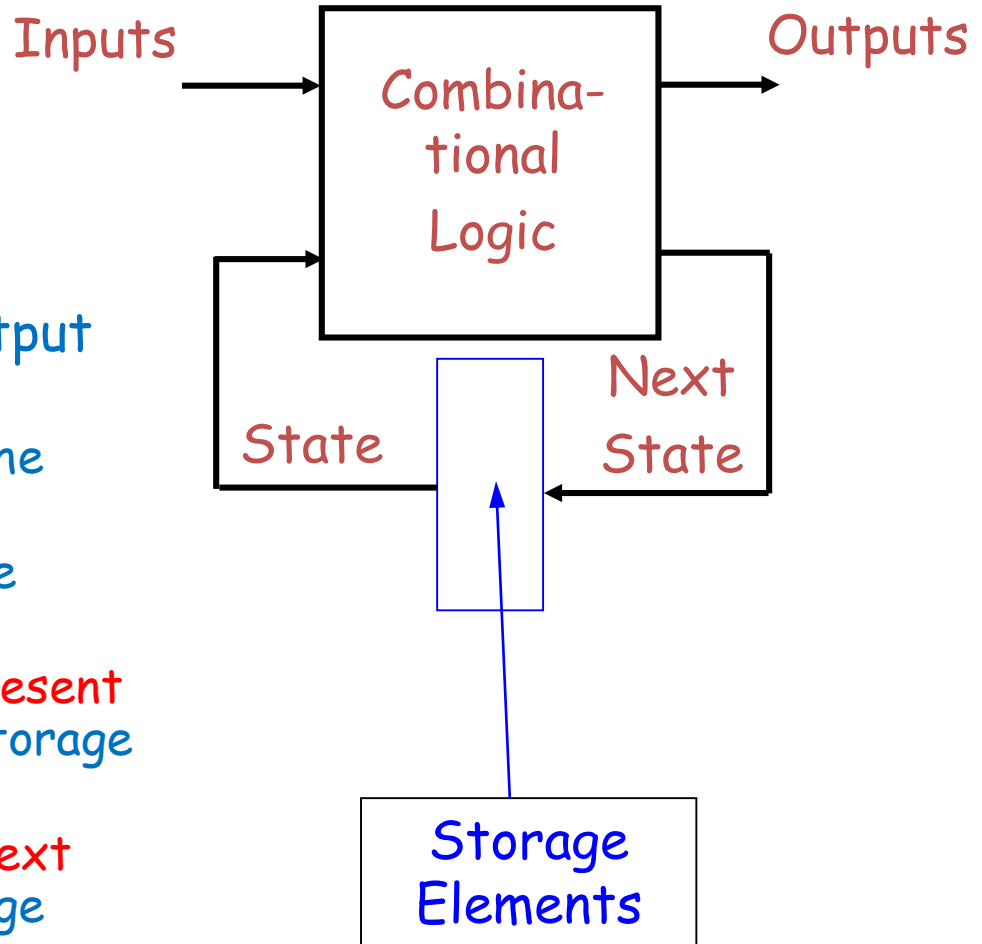


- ✓ 1-bit memory and 2 4-bit memory
- ✓ Only one full-adder!
- ✓  $n$  clocks (four in this case) to get the output
- ✓ The 1-bit memory defines the circuit state (0 or 1)

# Introduction to Sequential Circuits

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- ✓ A Sequential circuit contains:
- Storage elements: **Latches or Flip-Flops**
  - Combinational Logic that implements a multiple-output switching function:
    - **Inputs** are signals from the outside.
    - **Outputs** are signals to the outside.
    - Other inputs, **State or Present State**, are signals from storage elements.
    - The remaining outputs, **Next State** are inputs to storage elements.



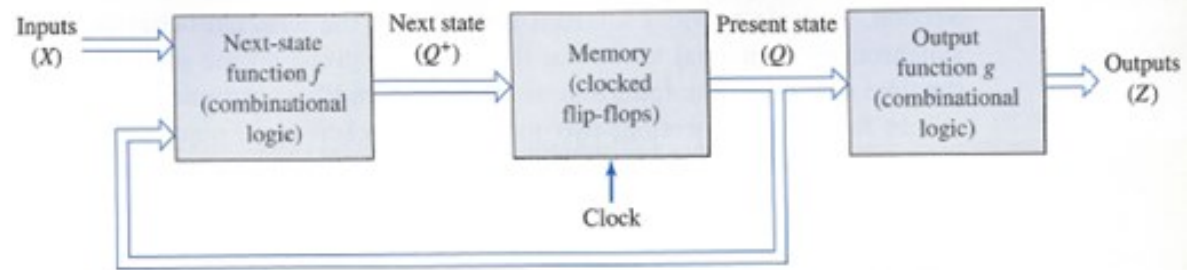
# Introduction to Sequential Circuits

Combinatorial Logic

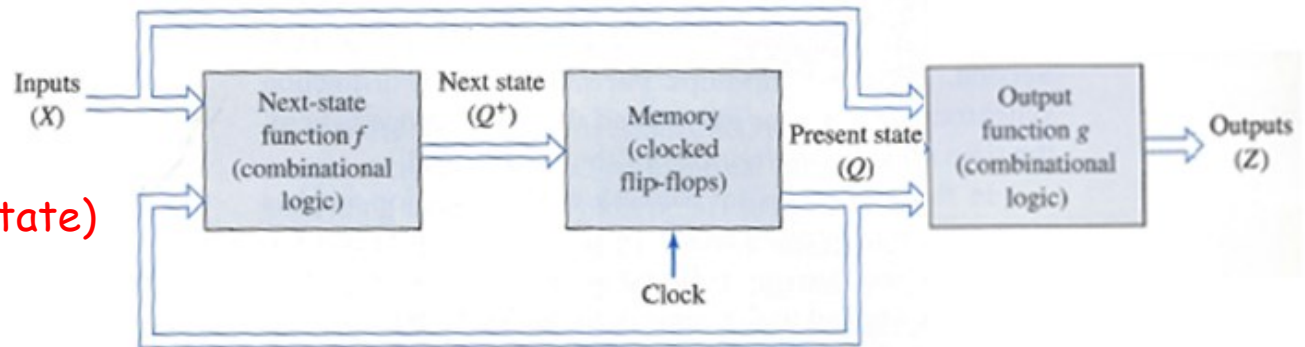
Next state function  $\text{Next State} = f(\text{Inputs}, \text{State})$

Output function, two types:

- **Moore**  
 $\text{Outputs} = h(\text{State})$



- **Mealy**  
 $\text{Outputs} = g(\text{Inputs}, \text{State})$



Output function type depends on specification and affects the design significantly

# Types of Sequential Circuits

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- ✓ Depends on the **times** at which:
  - **storage elements** observe **their inputs** and **change their state**
- ✓ **Synchronous**
  - Behavior is defined from knowledge of signals at **discrete** instances of time
  - Storage elements observe inputs and can change state only in relation to a timing signal (**clock pulses** from a **clock**)
- ✓ **Asynchronous**
  - Behavior is defined from knowledge of inputs at **any instant of time** and the order in **continuous time** in which inputs change
  - If clock just regarded as another input, **all circuits are asynchronous!** Nevertheless, the **synchronous abstraction** makes complex designs tractable!

# Discrete Event Simulation

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- ✓ In order to understand the time behavior of a sequential circuit we use discrete event simulation.
- ✓ Rules:
  - Gates modeled by an ideal (instantaneous) function and a fixed gate delay
  - Any change in input values is evaluated to see if it causes a change in output value
  - Changes in output values are scheduled for the fixed gate delay after the input change
  - At the time for a scheduled output change, the output value is changed along with any inputs it drives

# Simulated NAND Gate

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- ✓ Example: A 2-Input NAND gate with a 0.5 ns. delay:



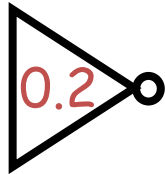
- ✓ Assume A and B have been 1 for a long time
- ✓ At time  $t=0$ , A changes to a 0 at  $t= 0.8$  ns, back to 1.

$t$ (ns)	A	B	F(I)	F	Comment
$-\infty$	1	1	0	0	$A=B=1$ for a long time
0	$1 \Rightarrow 0$	1	$1 \Leftarrow 0$	0	FI changes to 1
0.5	0	1	1	$1 \Leftarrow 0$	F changes to 1 after a 0.5 ns delay
0.8	$1 \Leftarrow 0$	1	$1 \Rightarrow 0$	1	FI changes to 0
0.13	1	1	0	$1 \Rightarrow 0$	F changes to 0 after a 0.5 ns delay

# Gate Delay Models

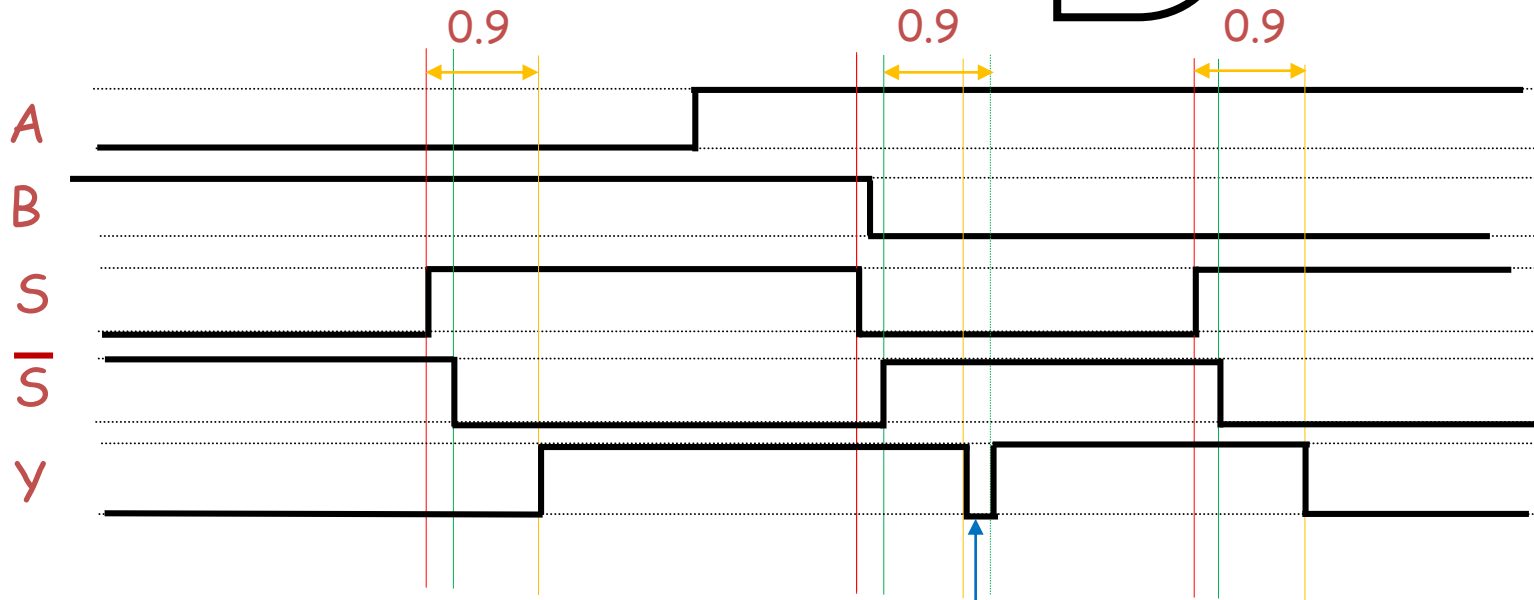
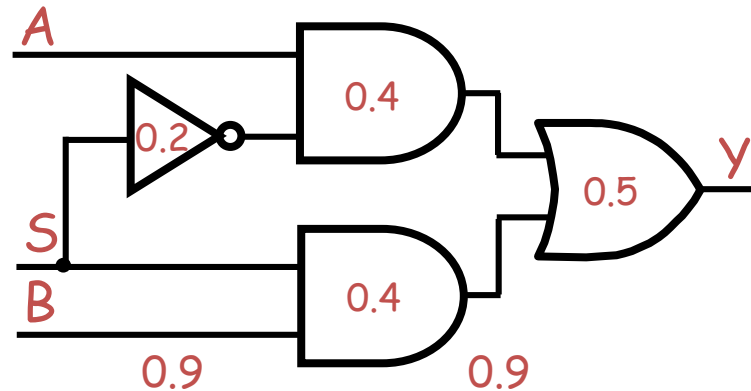
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- ✓ Suppose gates with delay  $n$  ns are represented for  $n = 0.2$  ns,  $n = 0.4$  ns,  $n = 0.5$  ns, respectively:

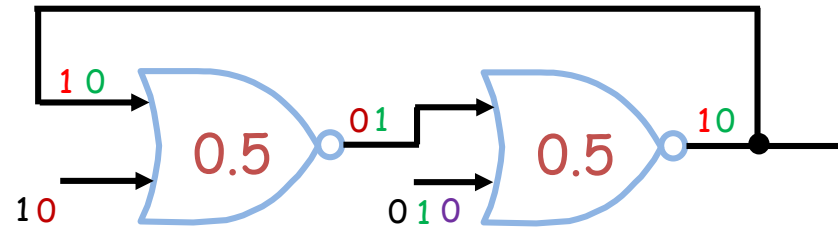


# Circuit Delay Model

- ✓ Consider a simple 2-input multiplexer:
- ✓ With function:
  - $Y = B$  for  $S = 1$
  - $Y = A$  for  $S = 0$



"computer glitch" is due to delay of inverter



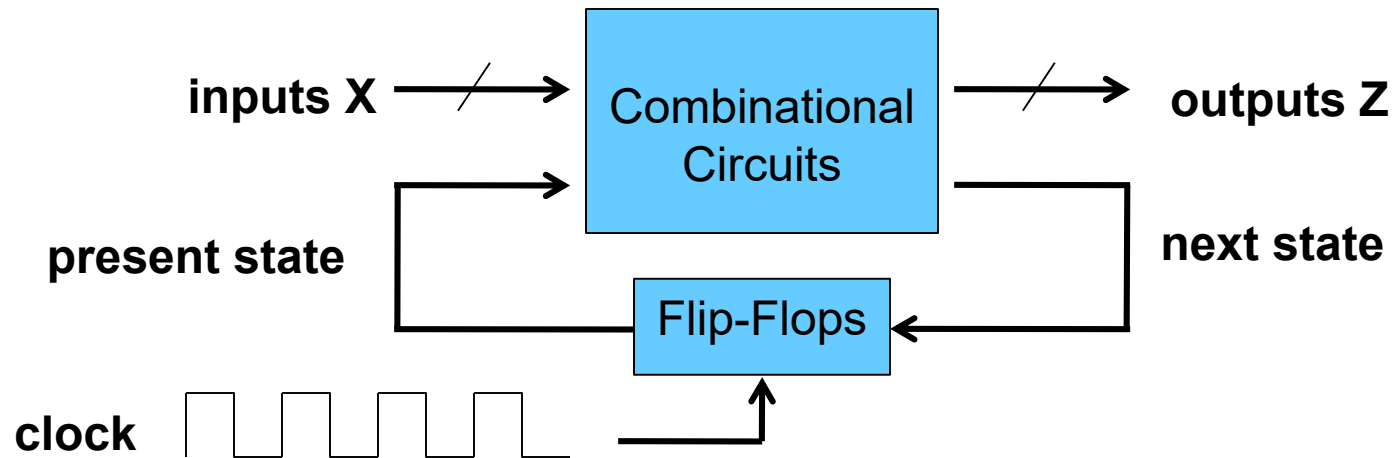
# Storage Elements (Memory)

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- ✓ A storage element can maintain a binary state (0,1) indefinitely, until is directed by an input signal to switch state,
- ✓ Main difference between storage elements:
  - Number of inputs they have
  - How the inputs affect the binary state
- ✓ Two main types:
  - Latches (level-sensitive)
  - Flip-Flops (edge-sensitive)
- ✓ Latches are useful in asynchronous sequential circuits
- ✓ Flip-Flops are synchronous sequential circuits, built with latches

# Synchronous Sequential Circuits

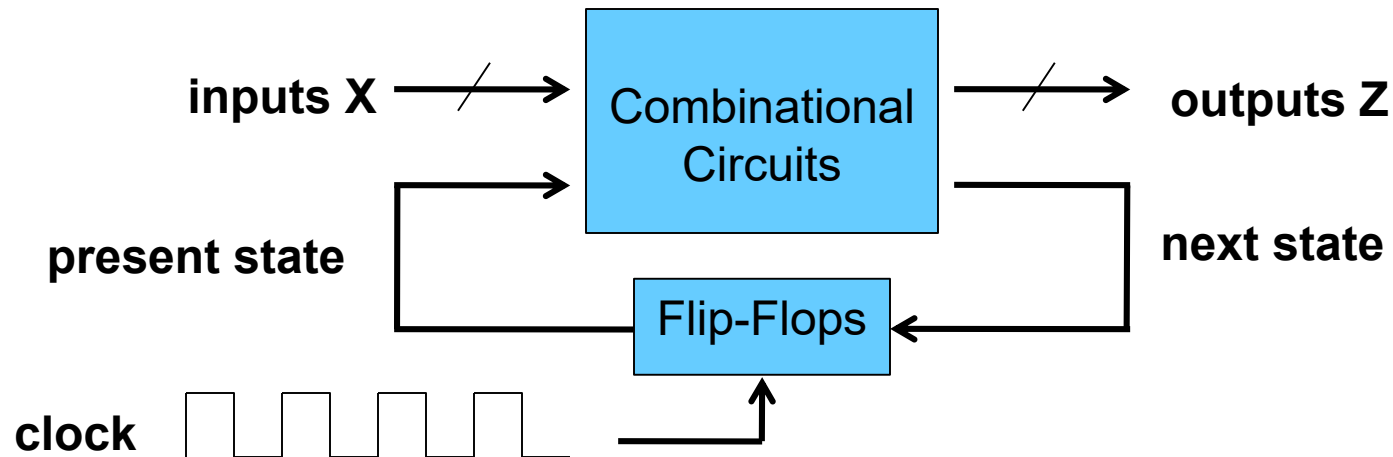
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- ✓ Synchronous circuits employ a synchronizing signal called **clock** (a periodic train of pulses; 0s and 1s)
- ✓ A clock determines **when** computational activities occur; the other signals determine **what** changes will occur

# Synchronous Sequential Circuits

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- ✓ The storage elements (memory) used in clocked sequential circuits are called **flip-flops**
  - Each flip-flop can store one bit of information 0,1
  - A circuit may use many flip-flops; together they define the circuit state
- ✓ **Flip-flops (memory/state) update only with the clock**

# Latches

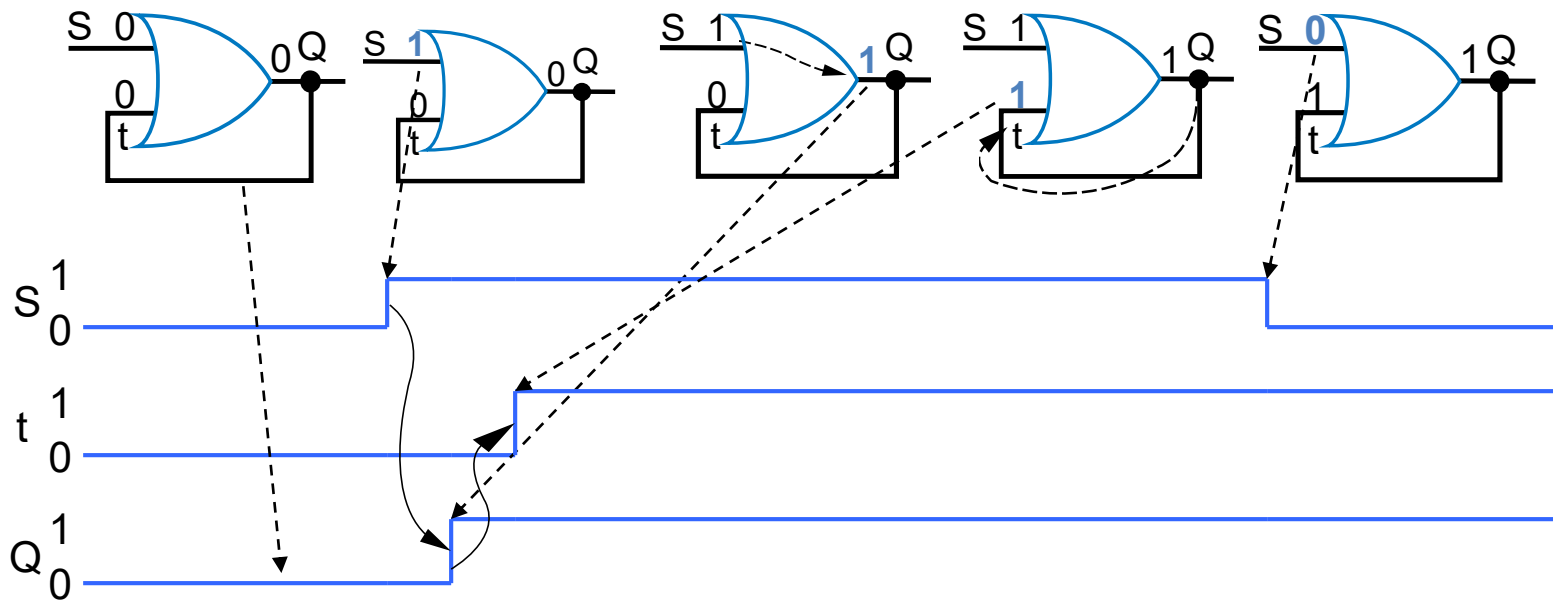
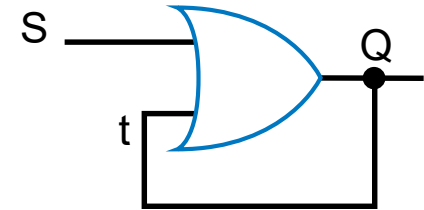
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- ✓ A latch is binary storage element
- ✓ Can store a 0 or 1
- ✓ The most basic memory
- ✓ Easy to build
  - Built with gates (NORs, NANDs, NOT)

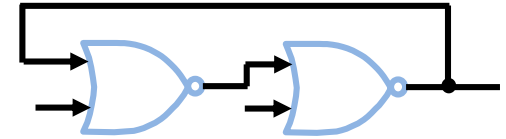
# First attempt at Bit Storage

✓ We need some sort of feedback

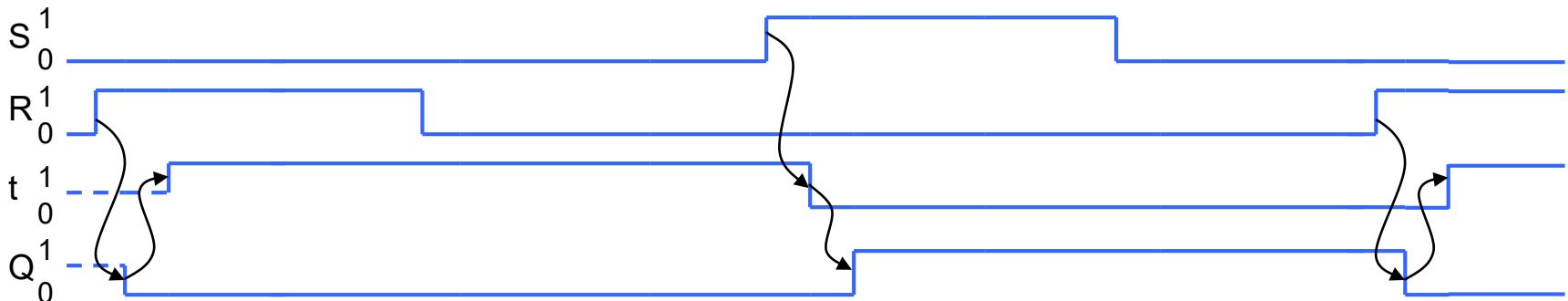
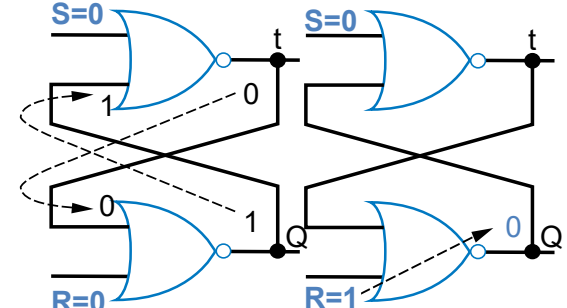
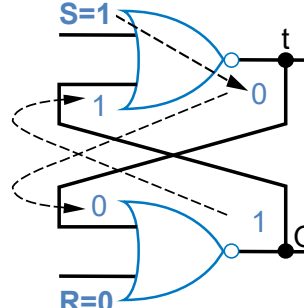
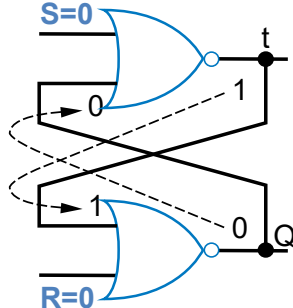
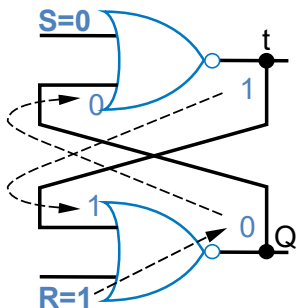
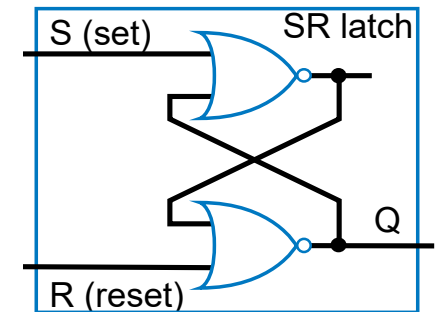
- Does circuit on the right do what we want?
  - No: Once Q becomes 1 (when  $S=1$ ), Q stays 1 forever - no value of S can bring Q back to 0



# Bit Storage Using an SR Latch



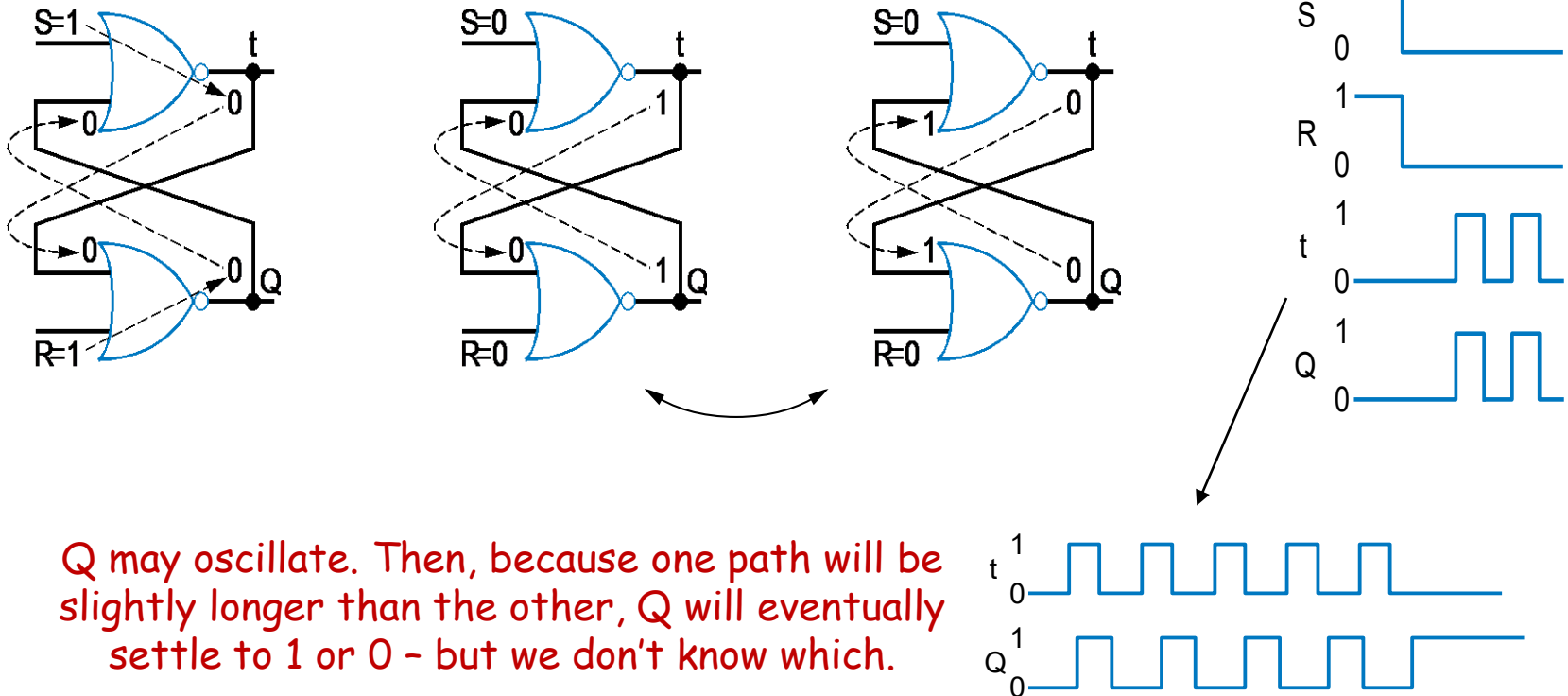
- ✓ Does the circuit to the right, with cross-coupled NOR gates, do what we want?



# Problem with SR Latch

## ✓ Problem

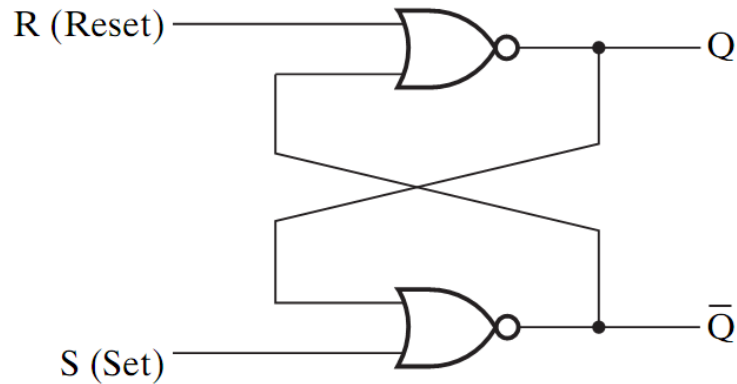
- If  $S=1$  and  $R=1$  simultaneously, we don't know what value  $Q$  will take



$Q$  may oscillate. Then, because one path will be slightly longer than the other,  $Q$  will eventually settle to 1 or 0 - but we don't know which.

# SR Latch

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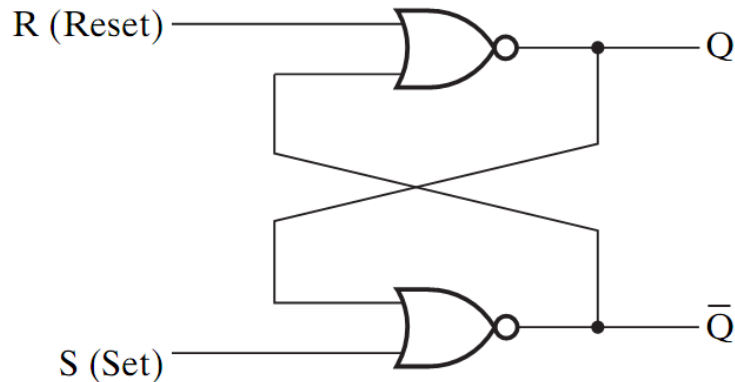
(a) Logic diagram

S	R	Q	$\bar{Q}$
1	0		
0	0		
0	1		
0	0		
1	1		

(b) Function table

✓ What does this circuit do?

# SR Latch



(a) Logic diagram

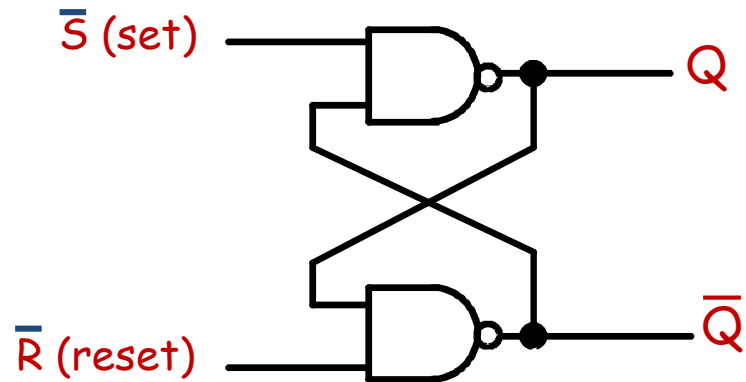
S	R	Q	$\bar{Q}$	
1	0	1	0	Set state
0	0	1	0	
0	1	0	1	Reset state
0	0	0	1	
1	1	0	0	Undefined

(b) Function table

- ✓ **Two states:** Set ( $Q = 1$ ) and Reset ( $Q = 0$ )
- ✓ When  $S=R=0$ ,  $Q$  remains the same,  $S=R=1$  is not allowed!
- ✓ Normally,  $S=R=0$  unless the state need to be changed: **MEMORY**
- ✓ **State of the circuit depends** not only on the current inputs, but also on the recent history of the inputs

# Basic (NAND) $\bar{S}$ - $\bar{R}$ Latch

- ✓ "Cross-Coupling" two NAND gates gives the  $\bar{S}$  -  $\bar{R}$  Latch:
- ✓ Which has the time sequence behavior:



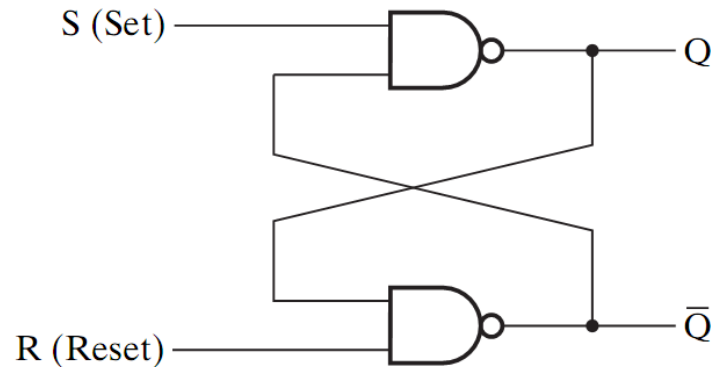
Time ↓

R	S	Q	$\bar{Q}$	Comment
1	1	?	?	Stored state unknown: 01/10
1	0	1	0	"Set" Q to 1
1	1	1	0	Now Q "remembers" 1
0	1	0	1	"Reset" Q to 0
1	1	0	1	Now Q "remembers" 0
0	0	1	1	Both go high      Unstable!

- ✓  $S = 0, R = 0$  is forbidden as input pattern

# S' R' Latch

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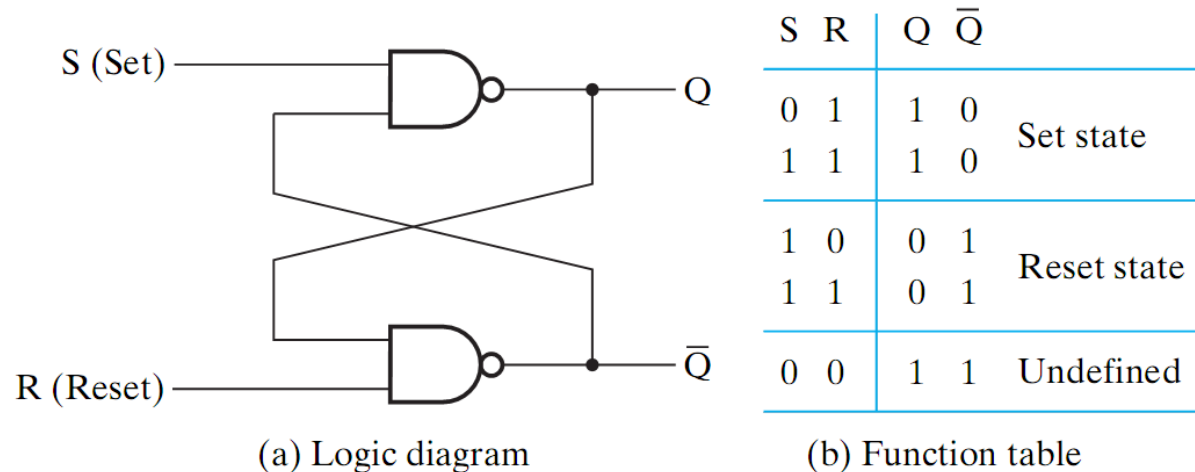
(a) Logic diagram

S	R	Q	$\bar{Q}$
0	1		
1	1		
1	0		
1	1		
0	0		

(b) Function table

✓ How about this circuit?

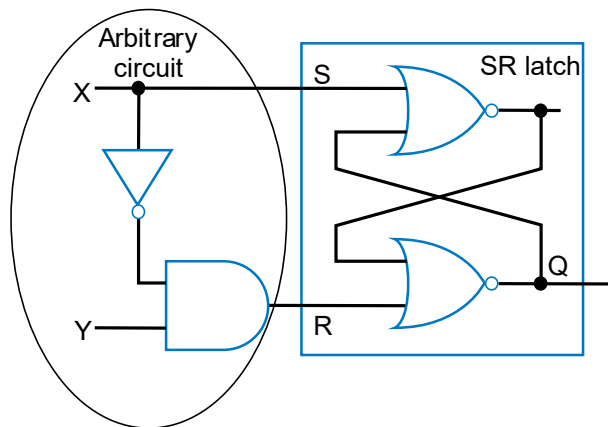
# S' R' Latch



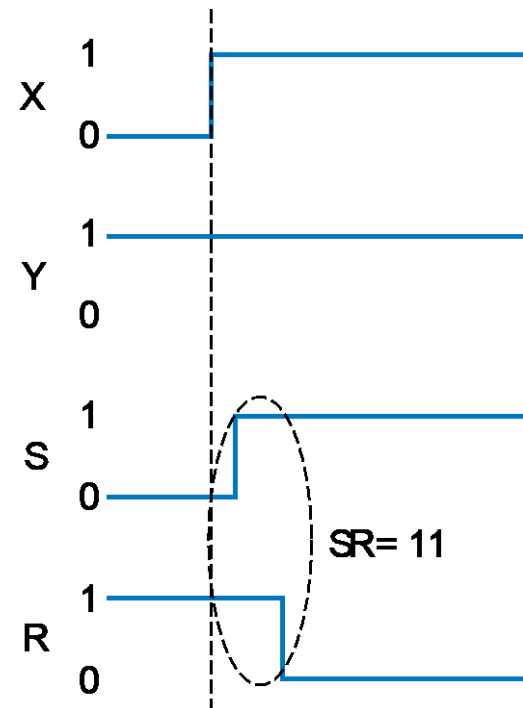
- ✓ Similar to SR latch (complemented)
- ✓ **Two states:** Set ( $Q = 0$ ) and Reset ( $Q = 1$ )
- ✓ When  $S=R=1$ ,  $Q$  remains the same
- ✓  **$S=R=0$  is not allowed!**

# Problem with SR Latch

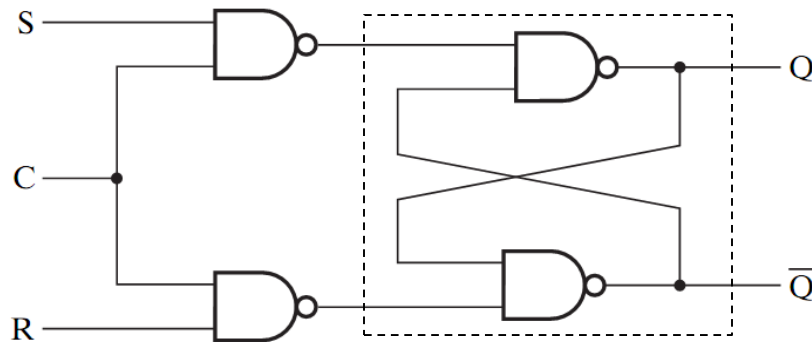
- ✓ The problem is not just a user pressing two buttons at same time
- ✓ Can also occur even if SR inputs come from a circuit that supposedly never sets  $S=1$  and  $R=1$  at same time
  - but does, due to different delays of different paths



The longer path from X to R than to S causes  $SR=11$  for short time - could be long enough to cause oscillation



# SR Latch with Clock



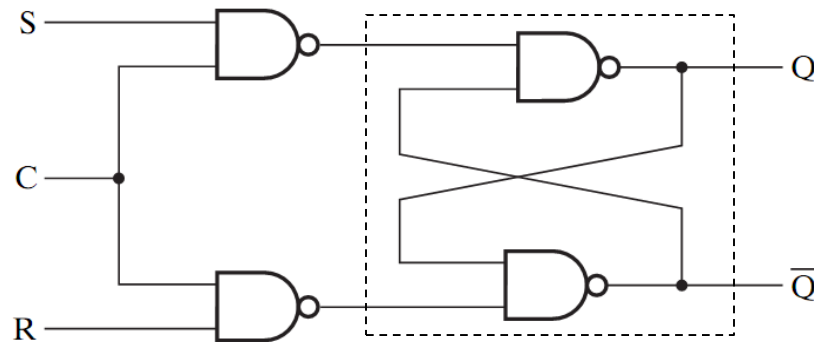
(a) Logic diagram

C	S	R	Next state of Q
0	X	X	No change
1	0	0	No change
1	0	1	Q = 0; Reset state
1	1	0	Q = 1; Set state
1	1	1	Undefined

(b) Function table

- ✓ An SR Latch can be modified to control **when** it changes an additional input signal Clock (C)
- ✓ When  $C=0$ , the S and R inputs have no effect on the latch
- ✓ When  $C=1$ , the inputs affect the state of the latch and possibly the output

# SR Latch with Clock (cont.)



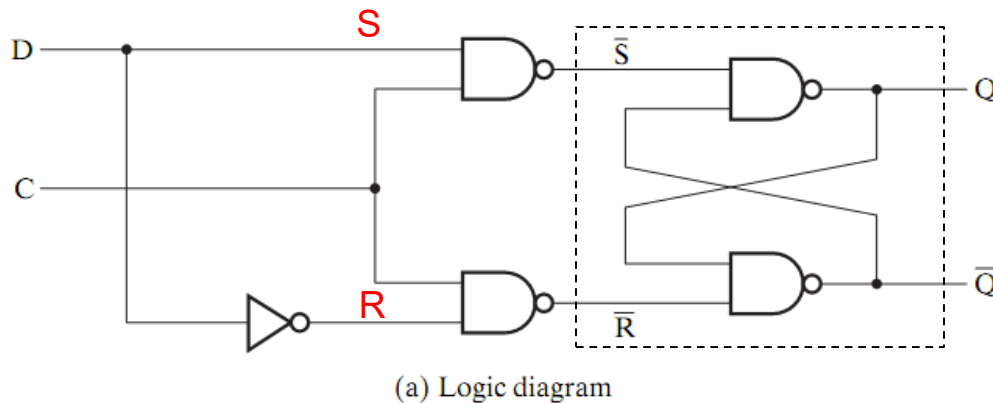
(a) Logic diagram

C	S	R	Next state of Q
0	X	X	No change
1	0	0	No change
1	0	1	Q = 0; Reset state
1	1	0	Q = 1; Set state
1	1	1	Undefined

(b) Function table

✓ How can we eliminate the undefined state?

# D Latch



(a) Logic diagram

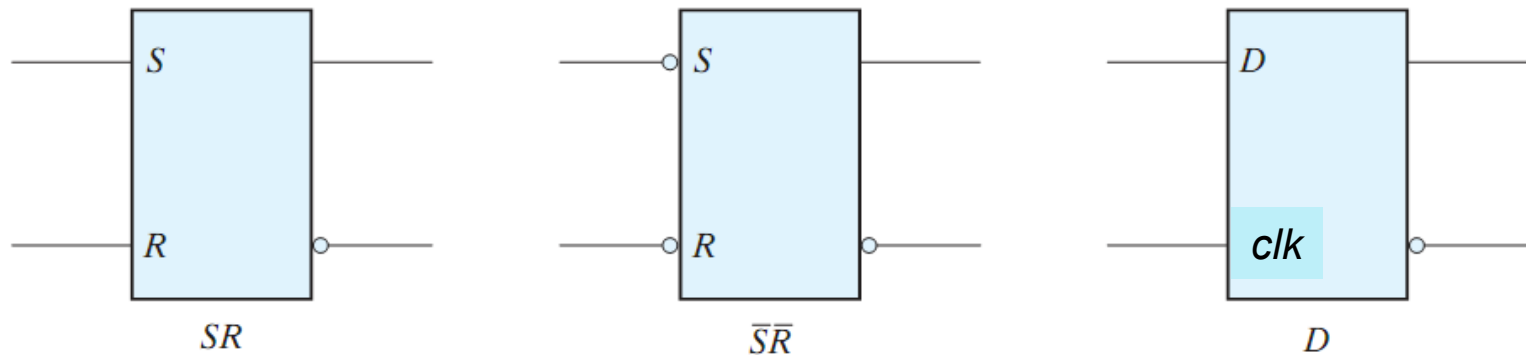
C	D	Next state of Q
0	X	No change
1	0	Q = 0; Reset state
1	1	Q = 1; Set state

(b) Function table

- ✓ Ensure S and R are never equal to 1 at the same time
- ✓ Add inverter
- ✓ Only one input (D)
  - D connects to S
  - $\bar{D}$  connects to R
- ✓ D stands for data
- ✓ Output follows the input when  $C = 1$ 
  - **Transparent**
- ✓ When  $C = 0$ , Q remains the same

# Graphic Symbols for Latches

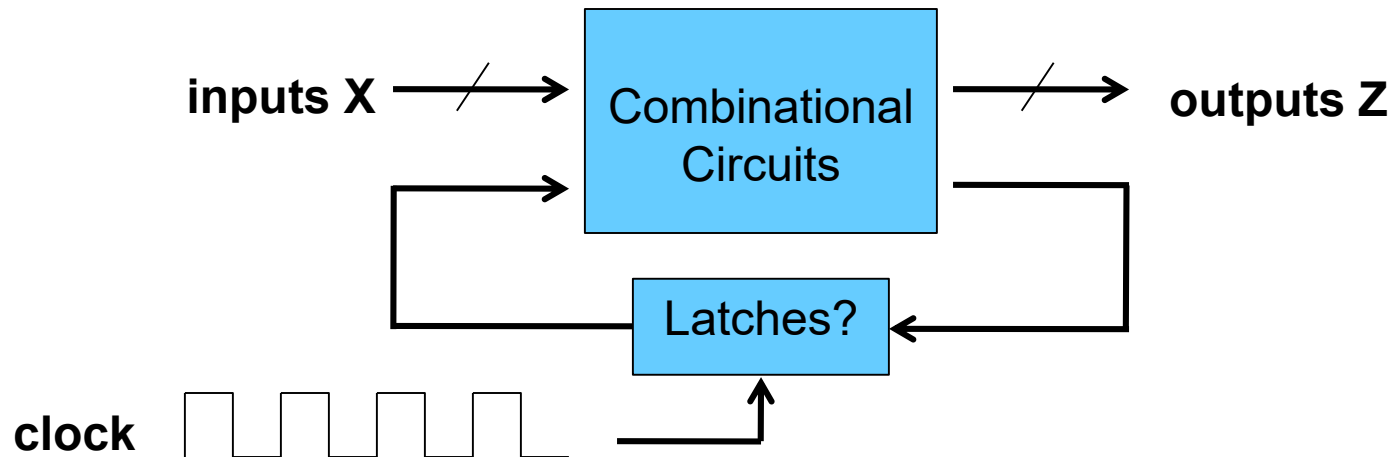
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- ✓ A latch is designated by a rectangular block with inputs on the left and outputs on the right
- ✓ One output designates the normal output, the other (with the bubble) designates the complement
- ✓ For  $\overline{SR}$  (SR built with NANDs), bubbles added to the input

# Problem with Latches

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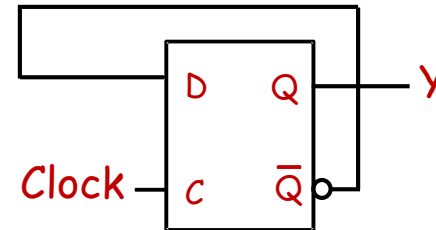


- ✓ What happens if Clock=1?
- ✓ **Problem:** A latch is **transparent**; state keep changing as long as the clock remains active
- ✓ Due to this uncertainty, latches can not be **reliably** used as storage elements.

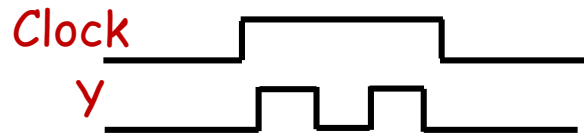
# The Latch Timing Problem

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- ✓ Consider the following circuit:



- ✓ Suppose that initially  $Y = 0$ .

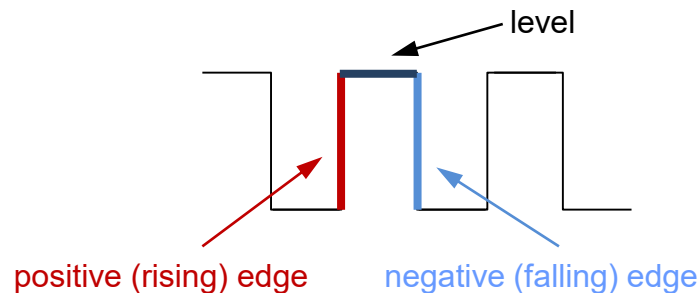


- ✓ As long as  $C = 1$ , the value of  $Y$  continues to change!
- ✓ The changes are based on the delay present on the loop.
- ✓ This behavior is clearly unacceptable.
- ✓ **Desired behavior:**  $Y$  changes **only once** per clock pulse

# Flip Flops

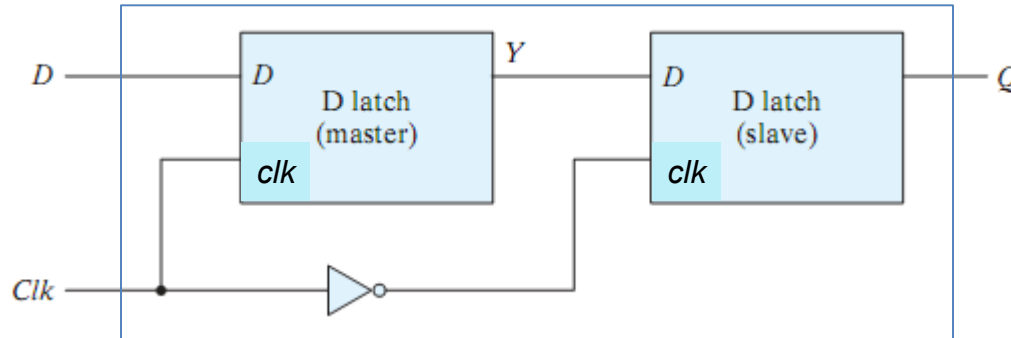
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- ✓ A flip-flop is a one bit memory similar to latches
- ✓ Solves the issue of latch transparency
- ✓ Latches are level sensitive memory element
  - Active when the clock = 1 (whole duration)
- ✓ Flip-Flops are edge-triggered or edge-sensitive memory element this solves the issue of latch transparency
  - Active only at transitions; i.e. either from  $0 \Rightarrow 1$  or  $1 \Rightarrow 0$



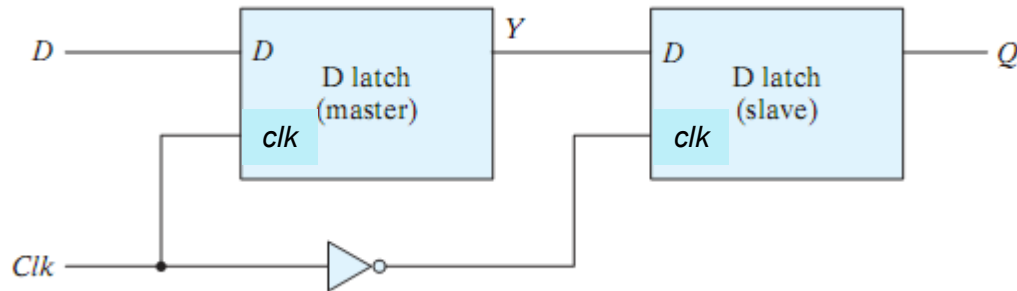
# Flip Flops

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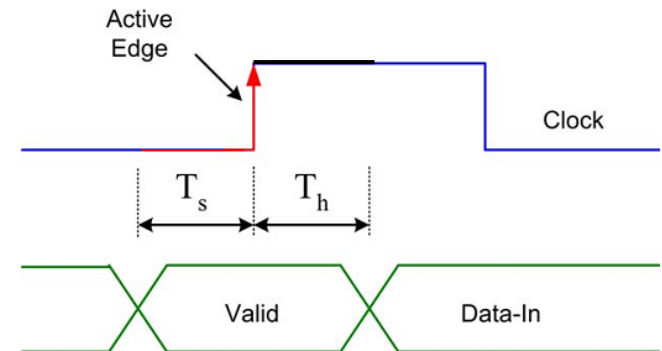
- ✓ A flip flop can be built using two latches in a **master-slave** configuration
  - A master latch receives external inputs
  - A slave latch receives inputs from the master latch
- ✓ Depending on the clock signal, only one latch is active at any given time
  - If clk **GO TO 1**, the master latch is enabled and the inputs are latched
  - if clk **GO TO 0**, the master is disabled and the slave is activated to generate the outputs

# Flip Flops



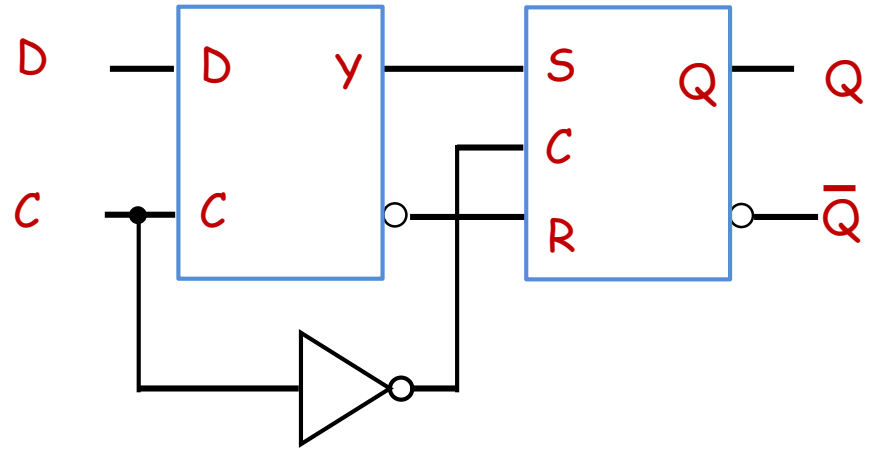
## ✓ Important Timing Considerations:

- **Setup Time ( $T_s$ ):** The minimum time during which  $D$  input must be maintained before the clock transition occurs.
- **Hold Time ( $T_h$ ):** The minimum time during which  $D$  input must not be changed after the clock transition occurs.

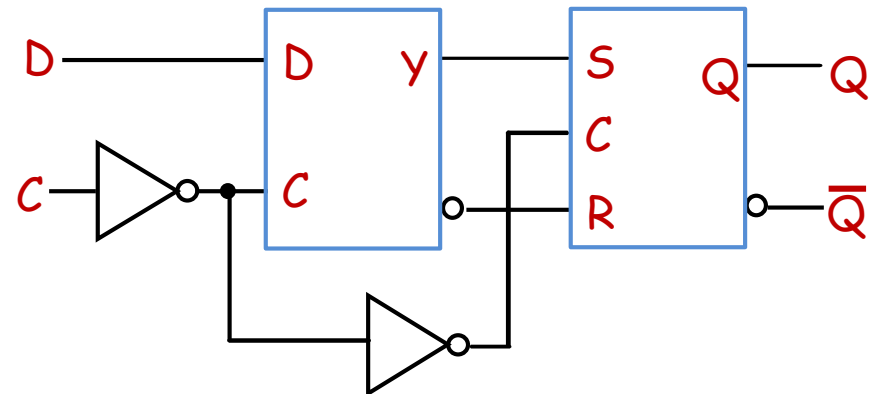


# Edge-Triggered D Flip-Flop

- ✓ The change of the D flip-flop output is associated with the negative edge at the end of the pulse. It is called a **negative-edge triggered flip-flop**

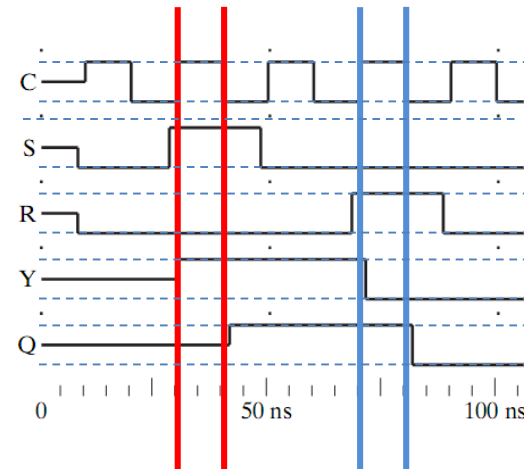
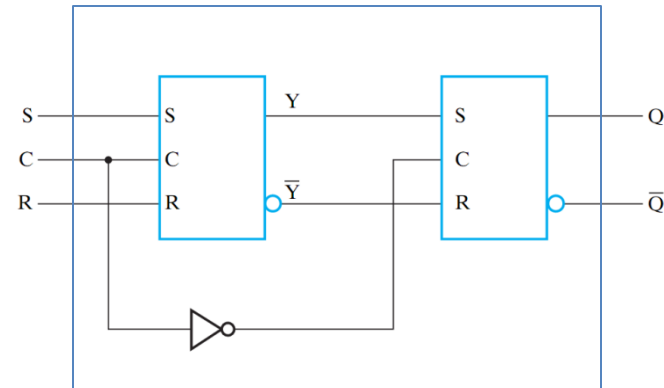


- ✓ **Positive-Edge Triggered D Flip-Flop** is formed by adding inverter to clock input. Q changes to the value on D applied at the positive clock edge within timing constraints to be specified

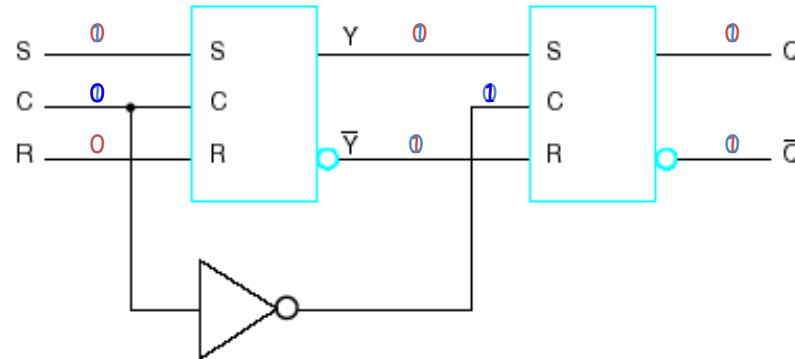


# SR Flip Flop

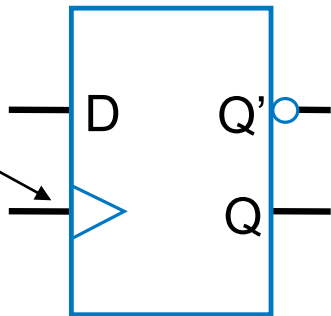
- ✓ Built using two latches (Master and Slave)
  - $C \Rightarrow 1$ , master is active
  - $C \Rightarrow 0$ , slave is active
- ✓ Data is entered on the rising edge of the clock pulse, but the output does not reflect the change until the falling edge of the clock pulse.
- ✓ Q is sampled at the falling edge



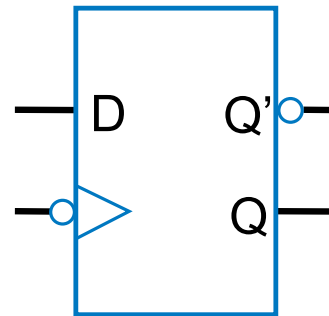
# SR flip-flop: execution example



The triangle means clock input, edge triggered

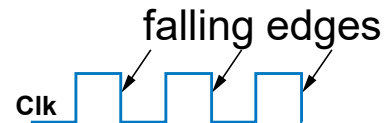
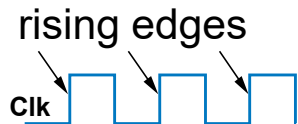


Symbol for rising-edge triggered D flip-flop



Internal design: Just invert servant clock rather than master

Symbol for falling-edge triggered D flip-flop



# D-Type Positive-Edge-Triggered Flip-Flop

✓ An efficient construction of an edge-triggered D flip-flop uses three SR latches. Two latches respond to the D and CLK inputs. The third latch provides the outputs.

✓ If the clock is 0, both the output signals of the input stage are 1 regardless of the data input D; the output latch is unaffected and it stores the previous state.

✓ When the clock signal changes from 0 to 1, only one of the output voltages (depending on the data signal) goes low and sets/resets the output latch:

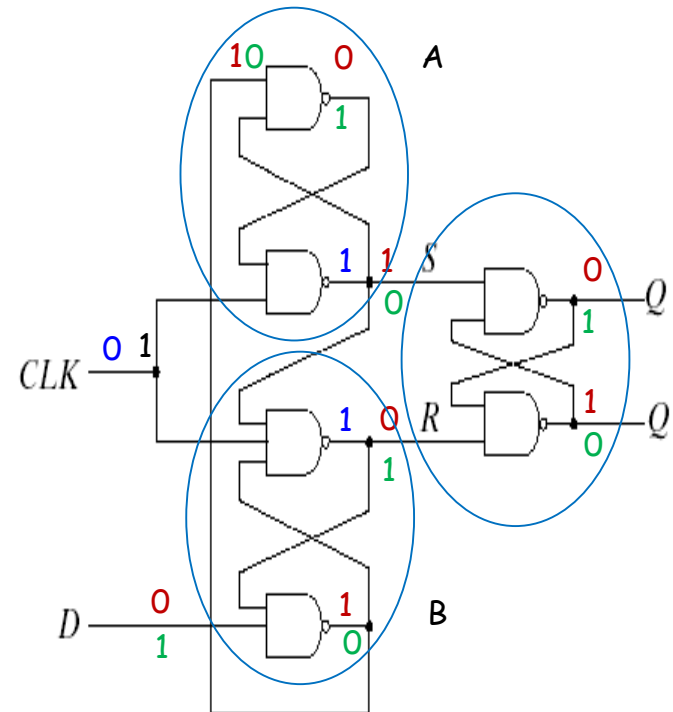
- if  $D = C = 0$ , the circuit will settle for a stable state  $C, D, R, S = 0011$ .

When C changes from 0 to 1, the state transitions will be  $0011 \Rightarrow 1011 \Rightarrow 1001$  that will cause output  $\Rightarrow 0$ ;

- if  $C = 0, D = 1$ , the circuit will settle for a stable state  $C, D, R, S = 0111$ .

When C changes from 0 to 1, the state transitions will be  $0111 \Rightarrow 1111 \Rightarrow 1110$  that will cause output  $\Rightarrow 1$ .

✓ The two states, 1000, and 1100 can never be reached as there are no combinations of  $R, S = 00$ .



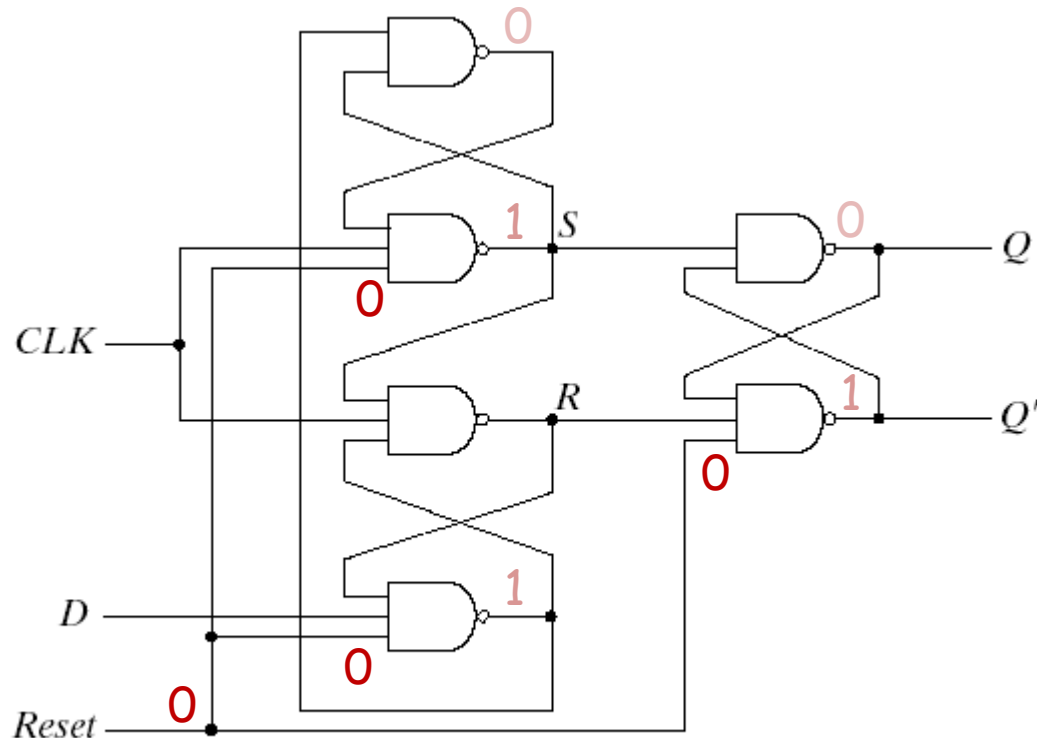
# Direct Inputs

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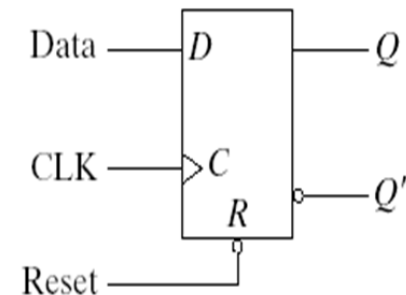
- ✓ Some flip-flops have asynchronous inputs that are used to force the flip-flop to a particular state independent of the clock.
- ✓ The input that sets the flip-flop to **1** is called **present or direct set**.
- ✓ The input that clears the flip-flop to **0** is called **clear or direct reset**.
- ✓ When power is turned on a digital system, the state of the flip-flops is **unknown**. The direct inputs are useful for bringing all flip-flops in the system to a known **starting state** prior to the clocked operation.

# D Flip-Flop with Asynchronous Reset

- ✓ A positive-edge-triggered D flip-flop with asynchronous reset



(a) Circuit diagram



(b) Graphic symbol

$R$	$C$	$D$	$Q$	$Q'$
0	X	X	0	1
1	$\uparrow$	0	0	1
1	$\uparrow$	1	1	0

(b) Function table

# Other Flip-Flop Types

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- ✓ Description of **J-K** and **T** flip-flops:
  - Behavior
  - Implementation
- ✓ Basic descriptors for understanding and using different flip-flop types:
  - **Characteristic table** - defines the **next state** of the flip-flop in terms of flip-flop **inputs** and **current state**
  - **Characteristic equation** - defines the **next state** of the flip-flop as a Boolean function of the flip-flop **inputs** and **the current state**
- ✓ Used in design
  - **Excitation table** - defines the flip-flop **input variable** values as function of the **current state** and **next state**

# J-K Flip-flop

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## ✓ Behavior:

- Same as S-R flip-flop with J analogous to S and K analogous to R
- Except that  $J = K = 1$  is allowed, and
  - For  $J = K = 1$ , the flip-flop changes to the opposite state

# T Flip-flop

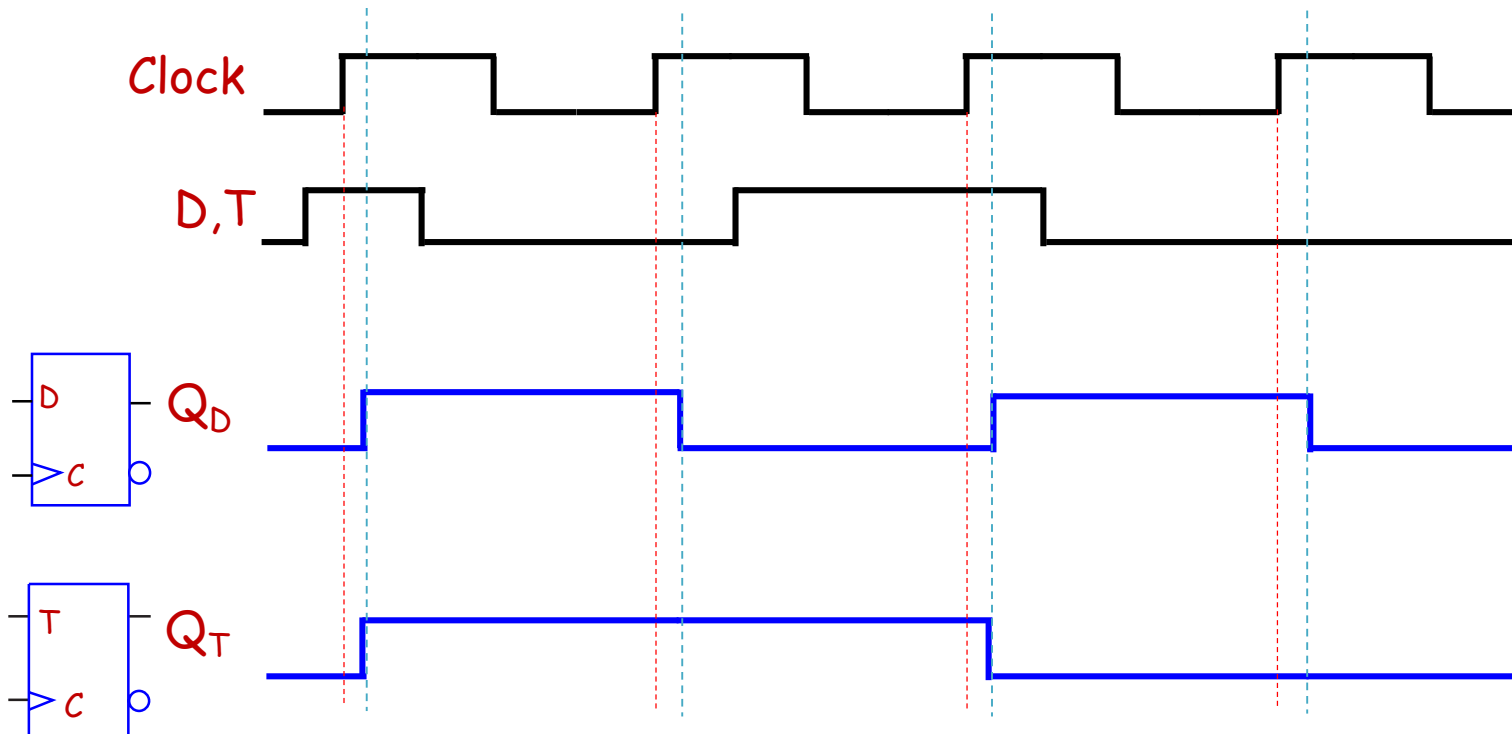
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- ✓ Behavior:
  - Has a single input T
    - For  $T = 0$ , no change to state
    - For  $T = 1$ , changes to opposite state
- ✓ Same as a J-K flip-flop with  $J = K = T$
- ✓ Cannot be initialized to a known state using the T input then: Reset is essential

# Flip-flop Behavior Example

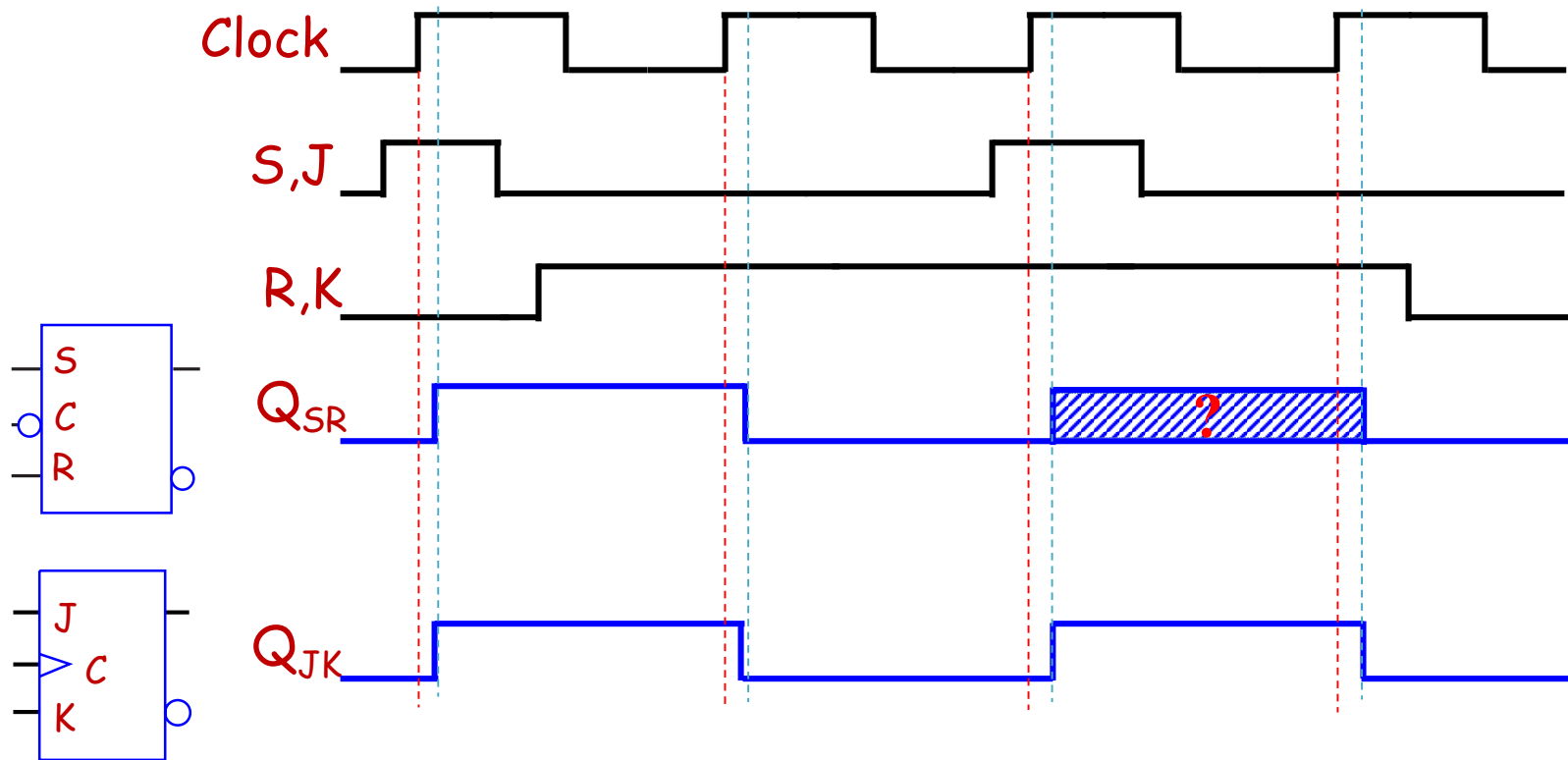
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- ✓ Use the characteristic tables to find the output waveforms for the flip-flops just **positive triggered**:



# Flip-Flop Behavior Example (continued)

- ✓ Use the characteristic tables to find the output waveforms for the flip-flops just **positive triggered**:



# Characteristic Tables

- ✓ A **characteristic table** defines the operation of a flip flop in a tabular form
- ✓ Next state is defined in terms of the current state and the inputs
  - $Q(t)$  refers to current state (**before** the clock arrives)
  - $Q(t+1)$  refers to next state (**after** the clock arrives)
- ✓ Similar to the truth table in combinational circuits

*Flip-Flop Characteristic Tables*

<b><i>JK Flip-Flop</i></b>			
<b><i>J</i></b>	<b><i>K</i></b>	<b><i>Q(t + 1)</i></b>	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

<b><i>D Flip-Flop</i></b>		
<b><i>D</i></b>	<b><i>Q(t + 1)</i></b>	
0	0	Reset
1	1	Set

<b><i>T Flip-Flop</i></b>		
<b><i>T</i></b>	<b><i>Q(t + 1)</i></b>	
0	$Q(t)$	No change
1	$Q'(t)$	Complement

# Characteristic Equations

✓ A characteristic equation defines the operation of a flip flop in an algebraic form

- For D-FF

$$Q(t+1) = D$$

- For JK-FF

$$Q(t+1) = J Q'(t) + K' Q(t)$$

- For T-FF

$$Q(t+1) = T \oplus Q(t)$$

- For SR-FF

$$Q(t+1) = S + R' Q(t)$$

*Flip-Flop Characteristic Tables*

**JK Flip-Flop**

<i>J</i>	<i>K</i>	$Q(t + 1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

**D Flip-Flop**

<i>D</i>	$Q(t + 1)$	
0	0	Reset
1	1	Set

**T Flip-Flop**

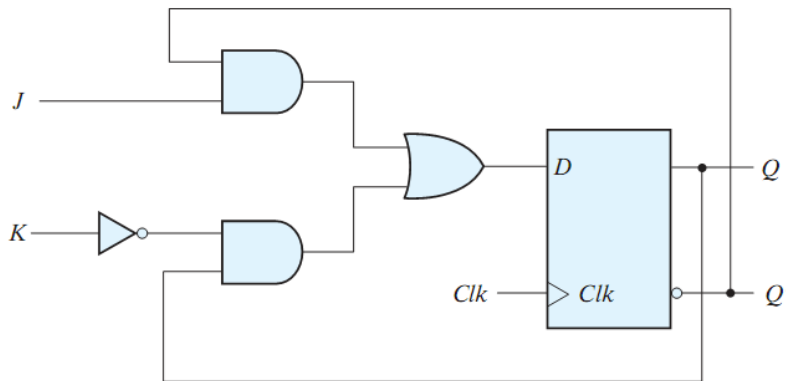
<i>T</i>	$Q(t + 1)$	
0	$Q(t)$	No change
1	$Q'(t)$	Complement

<i>S</i>	<i>R</i>	$Q(t+1)$	Operation
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Undefined

# Flip Flops Sheet

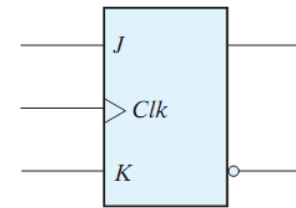
Characteristic Ta ble				Characteristic Equation	Excitation Table					
D		Q(t+1)	Operation	$Q(t+1)=D(t)$	Q(t+1)		D		Operation	
0		0	Reset		0		0		Reset	
1		1	Set		1		1		Set	
S	R	Q(t+1)	Operation	$Q(t+1)=S(t)+\overline{R}(t)Q(t)$	Q(t)	Q(t+1)	S	R	Operation	
0	0	$Q(t)$	No change		0	0	0	X	No change	
0	1	0	Reset		0	1	1	0	Set	
1	0	1	Set		1	0	0	1	Reset	
1	1	?	Undefined		1	1	X	0	No change	
J	K	Q(t+1)	Operation	$Q(t+1)=J(t)\overline{Q}(t)+\overline{K}(t)Q(t)$	Q(t)	Q(t+1)	J	K	Operation	
0	0	$Q(t)$	No change		0	0	0	X	No change	
0	1	0	Reset		0	1	1	X	Set	
1	0	1	Set		1	0	X	1	Reset	
1	1	$\overline{Q}(t)$	Complement		1	1	X	0	No Change	
	T	Q(t+1)	Operation	$Q(t+1)=T(t)\oplus Q(t)$	Q(t+1)		T		Operation	
	0	$Q(t)$	No change		$Q(t)$		0		No change	
	1	$\overline{Q}(t)$	Complement		$\overline{Q}(t)$		1		Complement	

# Flip Flops change

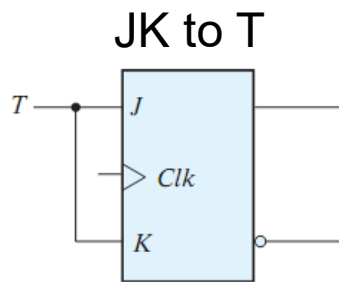


(a) Circuit diagram

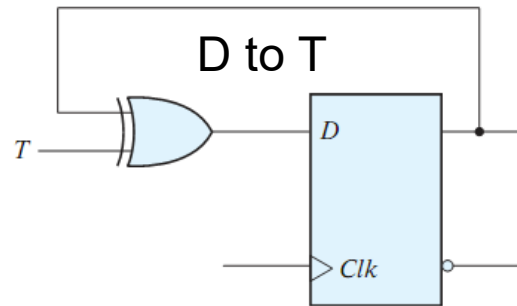
D to JK



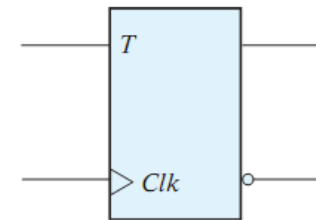
(b) Graphic symbol



(a) From JK flip-flop



(b) From D flip-flop

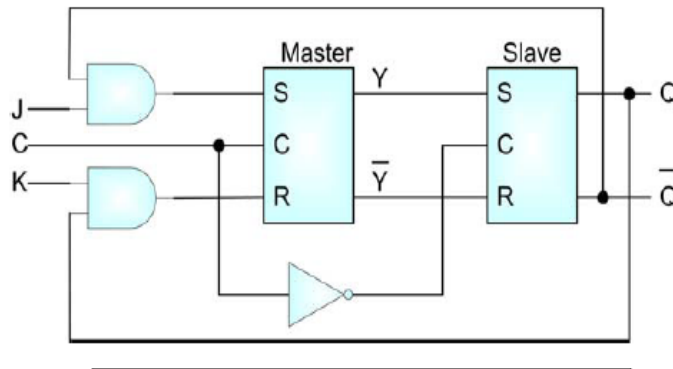


(c) Graphic symbol

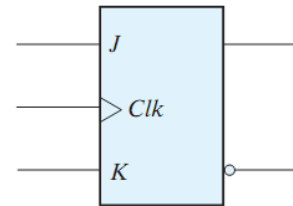
# Flip Flops change

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## JK Flip Flop built with SR latches



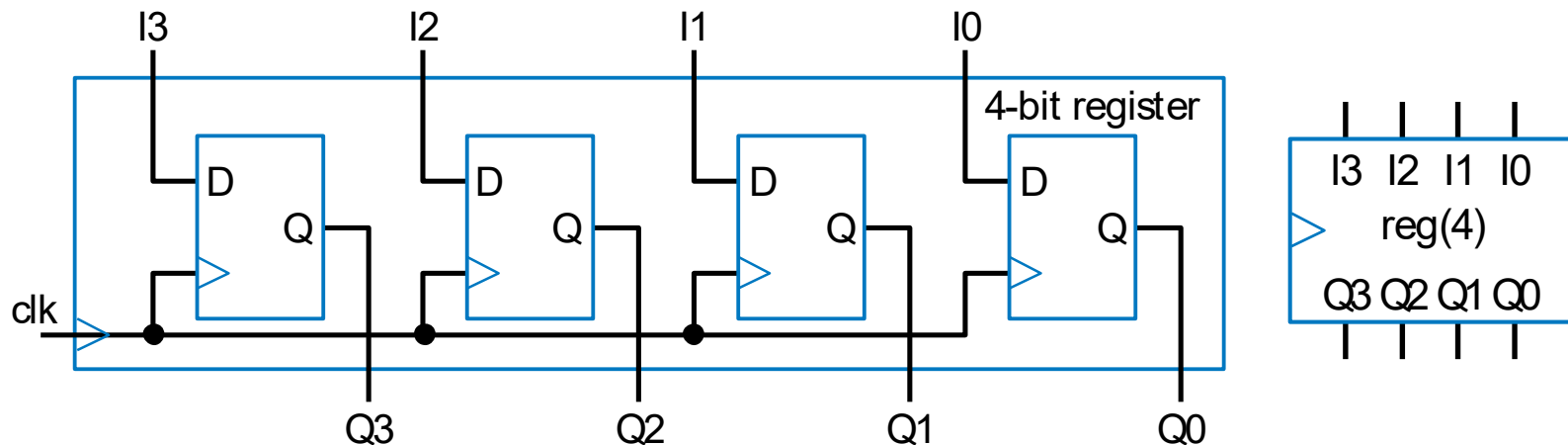
(a) Circuit diagram



(b) Graphic symbol

# Basic Register

- ✓ Typically, we store multi-bit items
  - e.g., storing a 4-bit binary number
- ✓ **Register** : multiple flip-flops sharing clock signal



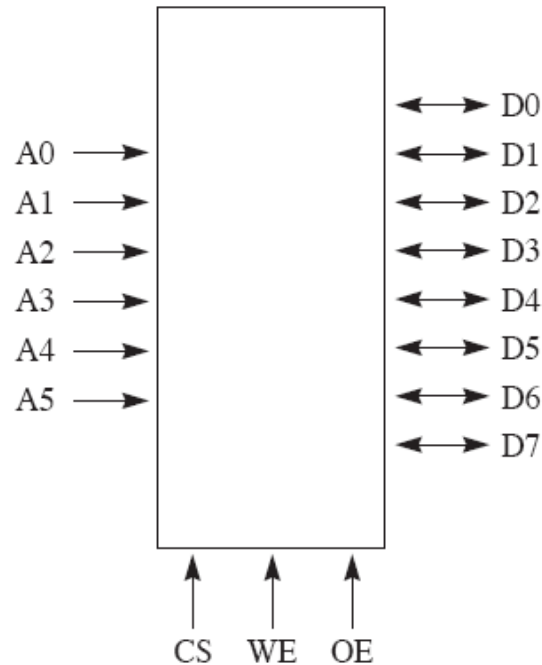
# Memory

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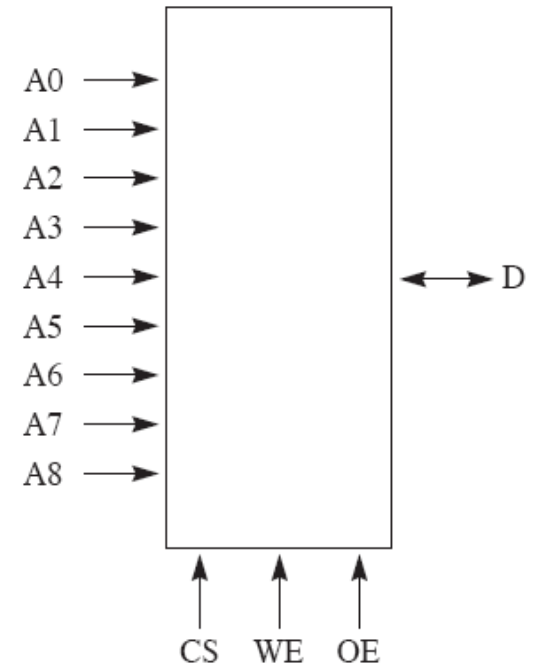
- ✓ Conceptually, main memory is just a big array of registers
- ✓ Input: address lines, control lines, bidirectional data lines
- ✓ Control signals:
  - CS: Chip select, to enable or select the memory chip
  - WE: Write enable, to write or store a memory word to the chip
  - OE: Output enable, to enable the output buffer to read a word from the chip

# Memory chips

Legenda:  
WE: write enable  
CS: chip select  
OE: output enable



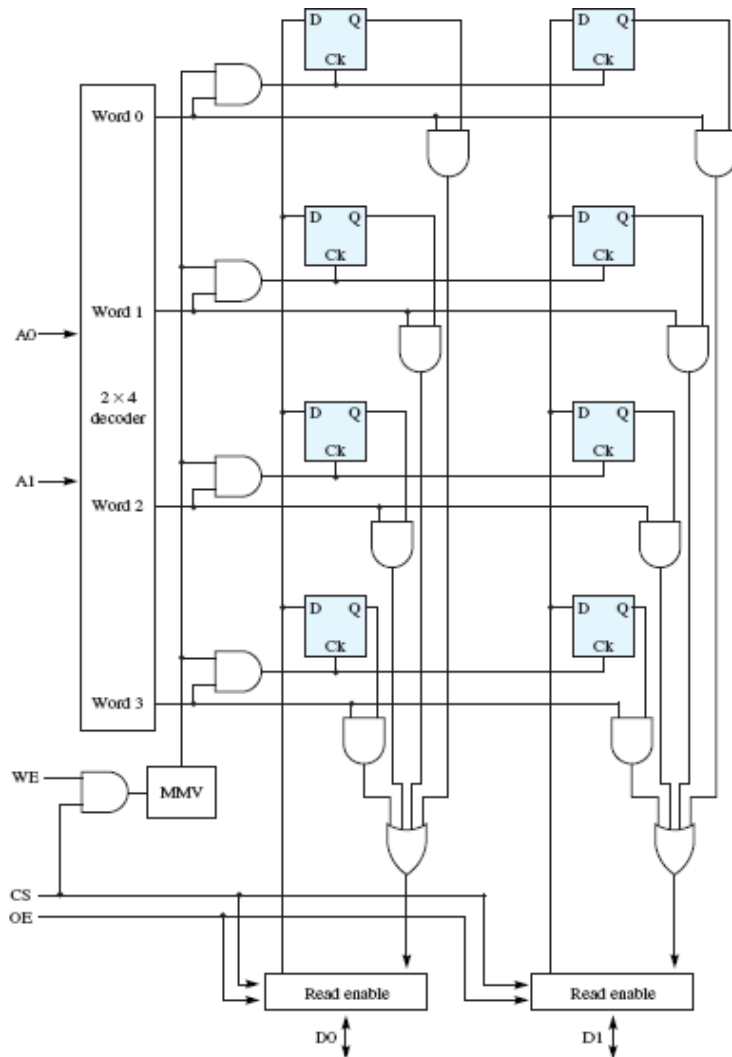
(a)  $64 \times 8$  bit memory chip.



(a)  $512 \times 1$  bit memory chip.

- ✓ **Storage capacity** of the two chips is identical (512 bits); left uses 8-bit word, right uses 1
- ✓ Generally, chip with  $2^n$  words has  $n$  address lines
- ✓ To store a word (**memory write**): a) select chip by setting CS to 1; b) put data and address on the bus and set WE to 1
- ✓ To retrieve a word (**memory read**): a) select chip by setting CS to 1; b) put address on the bus, set OE to 1, and read the data on the bus

# 4 x 2 memory chip



- ✓ Stores 4 words, 2-bit each. Each bit is D flip flop
- ✓ 2 address lines (A0, A1) & 2 data lines (D0, D1)
- ✓ MMV: monostable multivibrator
- ✓ Address lines drive 2 x 4 decoder
  - 1 output is 1, other 3 are 0
  - line with 1 signal selects row of D flip flops that make up word accessed by chip

# Closer look

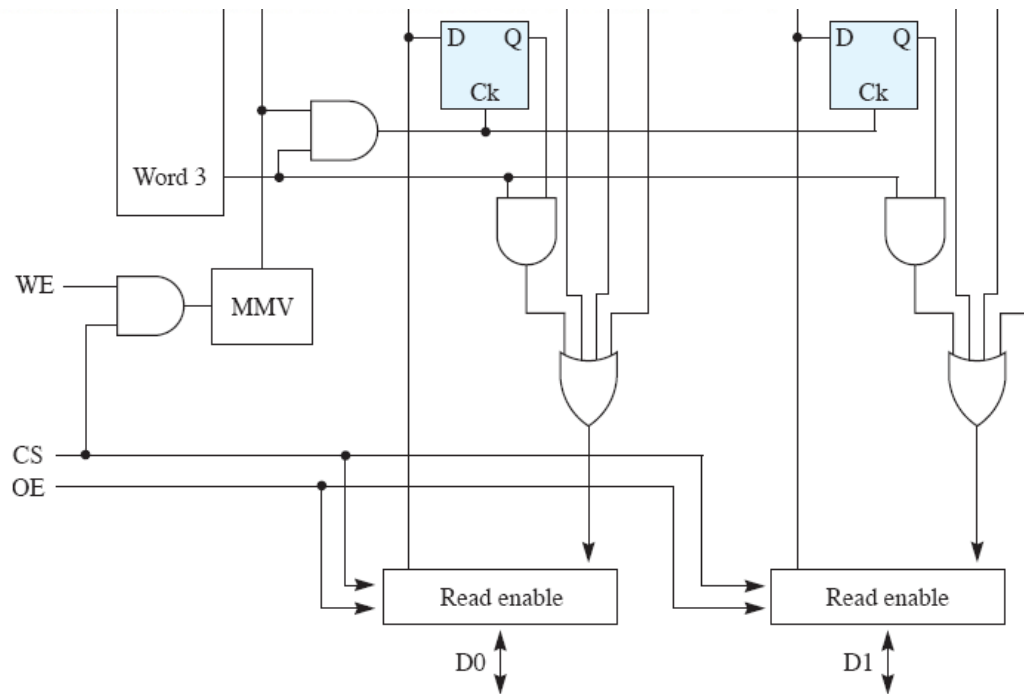
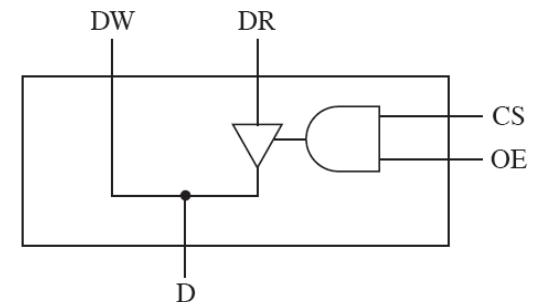


Diagram below shows implementation of **Read enable** box



CS	OE	Operation
0	×	Disconnected
1	0	Disconnected
1	1	Connect DR to D

Legenda:

WE: write enable

CS: chip select

OE: output enable

MMV: monostable multivibrator

✓ Normal modes:

- CS=0 (chip not selected)
- CS=1, WE=1, OE=0 (selected for write)
- CS=1, WE=0, OE=1 (selected for read)
- WE=1 & OE=1 not permitted

# Memory types: volatile

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- ✓ **SRAM**: Static random access memory
  - most closely resembles model we've seen
  - advantage: fast
  - disadvantage: large - several transistors required for each bit cell
- ✓ **DRAM**: Dynamic RAM
  - overcomes size problem of SRAM: one transistor, one capacitor per cell
  - advantage: high capacity
  - disadvantage: relatively slow because requires refresh operation

# Summary

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- ✓ In a sequential circuit, outputs depends on inputs and previous inputs
  - Previous inputs are stored as binary information into memory
  - The stored information at any time defines a state
  - Similarly, next state depends on inputs and present state
- ✓ Two types of sequential circuits: Synchronous and Asynchronous
- ✓ Two types of Memory elements: Latches and Flip-Flops.
- ✓ Flip-flops are built with latches
- ✓ A flip-flop is described using characteristic table/equation